edexcel

# Mark Scheme (Results)

## Summer 2016

## Pearson Edexcel GCE in Core Mathematics 2 (6664/01)

**Edexcel and BTEC Qualifications**

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

**Pearson: helping people progress, everywhere**

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2016
Publications Code 6664_01_1606_MS

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Core Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles).*

**Method mark for solving 3 term quadratic:**

### 1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

### 2. Formula

Attempt to use the correct formula (with values for a, b and c).

### 3. Completing the square

Solving $x^2 + bx + c = 0$ : $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

**Method marks for differentiation and integration:**

### 1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

### 2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **1.** | $r=\frac{3}{4},\ S_4=175$ | | |
| (a) **Way 1** | $\frac{a\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}}$ or $\frac{a\left(1-\frac{3}{4}^4\right)}{1-\frac{3}{4}}$ or $\frac{a\left(1-0.75^4\right)}{1-0.75}$ | Substituting $r=\frac{3}{4}$ or 0.75 and $n=4$ into the formula for $S_n$ | M1 |
| | $175=\frac{a\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}} \Rightarrow a=\frac{175\left(1-\frac{3}{4}\right)}{\left(1-\left(\frac{3}{4}\right)^4\right)}\ \left\{\Rightarrow a=\frac{\left(\frac{175}{4}\right)}{\left(\frac{175}{256}\right)}\Rightarrow\right\}\ \underline{a=64}$ * | Correct proof | A1* |
| | | | **[2]** |
| (a) **Way 2** | $a+a\left(\frac{3}{4}\right)+a\left(\frac{3}{4}\right)^2+a\left(\frac{3}{4}\right)^3$ | $a+a\left(\frac{3}{4}\right)+a\left(\frac{3}{4}\right)^2+a\left(\frac{3}{4}\right)^3$ | M1 |
| | $\frac{175}{64}a=175\ \left(\Rightarrow a=\frac{175}{\left(\frac{175}{64}\right)}\right)\Rightarrow \underline{a=64}$ * <br> or 2.734375$a$=175 $\Rightarrow \underline{a=64}$ | Correct proof | A1* |
| | | | **[2]** |
| (a) **Way 3** | $\{S_4=\}\ \frac{64\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}}$ or $\frac{64\left(1-\frac{3}{4}^4\right)}{1-\frac{3}{4}}$ or $\frac{64\left(1-0.75^4\right)}{1-0.75}$ | Applying the formula for $S_n$ with $r=\frac{3}{4}$, $n=4$ and $a$ as 64. | M1 |
| | $=175$ so $a=64$* | Obtains 175 with no errors seen and concludes $a=64$*. | A1* |
| | | | **[2]** |
| (b) | $\{S_\infty\}=\frac{64}{\left(1-\frac{3}{4}\right)}$; $=256$ | $S_\infty=\frac{(\text{their } a)}{1-\frac{3}{4}}$ or $\frac{64}{1-\frac{3}{4}}$ | M1; |
| | | 256 | A1cao |
| | | | **[2]** |
| (c) | $\{D=T_9-T_{10}=\}\ 64\left(\frac{3}{4}\right)^8-64\left(\frac{3}{4}\right)^9$ | Writes down either $"64"\left(\frac{3}{4}\right)^8$ or awrt 6.4 or $"64"\left(\frac{3}{4}\right)^9$ or awrt 4.8, using $a=64$ or their $a$ | M1 |
| | | A correct expression for the difference $\left(\text{i.e. } \pm\left(T_9-T_{10}\right)\right)$ using $a=64$ or their $a$. | dM1 |
| | $\left\{=64\left(\frac{3}{4}\right)^8\left(\frac{1}{4}\right)=1.6018066...\right\}=\underline{1.602}$ (3dp) | 1.602 or $-1.602$ | A1 cao |
| | | | **[3]** |
| | | | **7** |

| | | Question 1 Notes |
|---|---|---|
| **1.** (a) | | **Allow invisible brackets around fractions throughout all parts of this question**. |
| | **M1** | There are three possible methods as described above. |
| | **A1** | Note that this is a "show that" question with a printed answer.<br>In **Way 1** this mark **usually** requires $a = p/q$ where $p$ and $q$ may be unsimplified brackets from the formula (or could be 11200/175 for example) as an intermediate step before the conclusion $a = 64$.<br>Exceptions include $a = 175/4 * 256/175$ i.e. multiplication by reciprocal rather than division or 175 = 175$a$/64 followed by the obvious $a = 64$ These also get A1<br>In "reverse" methods such as **Way 3** we need a conclusion "so $a = 64$" or some implication that their argument is reversible. Also a conclusion can be implied from a <u>preamble</u>, eg: "If I assume $a = 64$ then find $S$= 175 as given this implies $a = 64$ as required"<br>This is a show that question and there should be no loss of accuracy.<br>In all the methods **if** decimals are used there should **not be rounding**.<br>If 0.68359375 appears this is correct. If it is rounded it would not give the exact answer.<br>$64(1-0.31640625)$ or 43.75 are each correct – if they are rounded then treat this as incorrect<br>e.g. **Way 3: "**43.75/0.25 = 175 so $a = 64$ is A1" but **"**43/0.25 = 175 so $a = 64$ is A0" and "44/0.25 = 175 so a = 64 is A0"<br>Yet another **variant on Way 3**: take a=64 then find the next 3 terms as 48, 36, 27 then add 64+48+36+27 to get 175. Again need conclusion that $a = 64$ or some implication that their argument is reversible. Otherwise M1 A0 |
| (b) | **M1** | $S_\infty = \frac{64}{1-\frac{3}{4}}$ or $\frac{\text{(their } a \text{ found in part } (a))}{1-\frac{3}{4}}$ |
| | **A1** | 256 cao |
| (c) | **NB** | Using **Sum of 10 terms** minus **Sum of 9 terms** is NOT a misread Scores **M0M0A0** |
| | **M1** | Can be **implied.** Writes down either $64\left(\frac{3}{4}\right)^8$ **or** $64\left(\frac{3}{4}\right)^9$,<br>using $a = 64$ (or their $a$ found in part (a)). |
| | **Note** | Ignore candidate's labelling of terms. |
| | **Note** | $64\left(\frac{3}{4}\right)^8 = 6.407226563...$ and $64\left(\frac{3}{4}\right)^9 = 4.805419922\ldots$ |
| | **dM1** | **This is dependent on previous M mark and can be implied.** Either<br>$64\left(\frac{3}{4}\right)^8 - 64\left(\frac{3}{4}\right)^9$ or $64\left(\frac{3}{4}\right)^9 - 64\left(\frac{3}{4}\right)^8$ or awrt 6.4 – awrt 4.8, using $a = 64$ (or their $a$ from part (a))<br>. |
| | **Note** | 1st M1 and 2nd M1 can be implied by the value of their<br>$\text{difference} = \text{"their } a \text{ found in part (a)"} \times \frac{3^8}{4^9} \approx \frac{\text{"their } a \text{ found in part (a)"}}{40}$ |
| | **Note** | Either $64\left(\frac{3}{4}\right)^9 - 64\left(\frac{3}{4}\right)^{10}$ or $64\left(\frac{3}{4}\right)^{10} - 64\left(\frac{3}{4}\right)^9$ is 1st M1, 2nd M0. |
| | **A1** | 1.602 or $-1.602$ **cao (This answer with no working is M1M1A1) But 1.6 with no working is M0M0A0** |
| | **Note** | $\left\{D = \frac{1}{4}T_9 \Rightarrow\right\}$ $D = \frac{1}{4}(64)\left(\frac{3}{4}\right)^8$ is 1st M1, 2nd M1 |
| | **Special case** | Obtains awrt 6.4, then obtains awrt 4.8 but rounds to 6 – 5 when subtracting – award M1M1A0 |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $y = 8 - 2^{x-1}$, $0 \leqslant x \leqslant 4$ | | |
| **2.** (a) | 7 | 7 | B1 **cao** |
| | | | **[1]** |
| (b) | $\left(\int_0^4 \left(8 - 2^{x-1}\right) dx \approx\right) \frac{1}{2} \times 1; \times \underline{\left\{7.5 + 2\left(\text{"their 7"} + 6 + 4\right) + 0\right\}}$ | Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$ | B1; |
| | | <u>For structure of trapezium rule $\{............\}$</u> for a candidate's $y$-ordinates. | <u>M1</u> |
| | $\left\{= \frac{1}{2} \times 41.5\right\} = 20.75$ o.e. | 20.75 | A1 **cao** |
| | | | **[3]** |
| (c) | Area $(R) = \text{"20.75"} - \frac{1}{2}(7.5)(4)$ | | M1 |
| | $= 5.75$ | 5.75 | A1 **cao** |
| | | | **[2]** |
| | | | **6** |

**Question 2 Notes**

| | | |
|---|---|---|
| (a) | **B1** | For 7 only |
| (b) | **B1** | For using $\frac{1}{2} \times 1$ or $\frac{1}{2}$ or equivalent. |
| | **M1** | Requires the correct $\{......\}$ bracket structure. It needs the 7.5 stated but the 0 may be omitted. The inner bracket needs to be multiplied by 2 and to be the summation of the remaining $y$ values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed ( An extra repeated term forfeits the M mark however (unless it is 0)). M0 is awarded if values used in brackets are $x$ values instead of $y$ values |
| | **A1** | For 20.75 or fraction equivalent e.g. $20\frac{3}{4}$ *or* $\frac{83}{4}$ |
| | **Note** | **NB: Separate trapezia may be used** : B1 for 0.5, M1 for 1/2 $h(a + b)$ used 3 or 4 times Then A1 as before. |
| | **Special case**: | Bracketing mistake $0.5 \times (7.5 + 0) + 2($ their $7 + 6 + 4)$ scores B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 37.75 usually indicates this error. |
| | **Common error:** | Many candidates use $\frac{1}{2} \times \frac{4}{5}$ and score B0 Then they proceed with $\underline{\left\{7.5 + 2\left(\text{"their 7"} + 6 + 4\right) + 0\right\}}$ and score M1 This usually gives 16.6 for B0M1A0 |
| (c) | **M1** | their answer to (b) $-$ area of triangle with base 4 and height 7.5 or alternative correct method e.g. their answer to (b) $- \int_0^4 \left(7.5 - \frac{7.5}{4}x\right) dx$ (Even if this leads to a negative answer) This may be implied by a correct answer or by an answer where they have subtracted 15 from their answer to part (b). Must use answer to part (b). |
| | **A1** | 5.75 or fraction equivalent e.g. $5\frac{3}{4}$ *or* $\frac{23}{4}$ |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3.** | $P(7, 8)$ and $Q(10, 13)$ | | |
| (a) | $\{PQ =\}\ \sqrt{(7-10)^2+(8-13)^2}$ or $\sqrt{(10-7)^2+(13-8)^2}$ | Applies distance formula. Can be implied. | M1 |
| | $\{PQ\} = \sqrt{34}$ | $\sqrt{34}$ or $\sqrt{17}.\sqrt{2}$ | A1 |
| | | | **[2]** |
| (b) **Way 1** | $(x-7)^2+(y-8)^2 = 34$ $\left(\text{or } \left(\sqrt{34}\right)^2\right)$ | $(x \pm 7)^2 + (y \pm 8)^2 = k$, where $k$ is a positive value. | M1 |
| | | $(x-7)^2+(y-8)^2 = 34$ | A1 oe |
| | | | **[2]** |
| (b) **Way 2** | $x^2+y^2-14x-16y+79=0$ | $x^2+y^2 \pm 14x \pm 16y + c = 0$, where $c$ is any value $< 113$. | M1 |
| | | $x^2+y^2-14x-16y+79=0$ | A1 oe |
| | | | **[2]** |
| (c) **Way 1** | $\{\text{Gradient of radius}\} = \frac{13-8}{10-7}$ or $\frac{5}{3}$ | This must be seen or implied in part (c). | B1 |
| | Gradient of tangent $= -\frac{1}{m}\left(= -\frac{3}{5}\right)$ | Using a perpendicular gradient method on their gradient. So Gradient of tangent $= -\frac{1}{\text{gradient of radius}}$ | M1 |
| | $y - 13 = -\frac{3}{5}(x-10)$ | $y - 13 = (\text{their changed gradient})(x-10)$ | M1 |
| | $3x + 5y - 95 = 0$ | $3x + 5y - 95 = 0$ o.e. | A1 |
| | | | **[4]** |
| (c) **Way 2** | $2(x-7) + 2(y-8)\frac{dy}{dx} = 0$ | Correct differentiation (or equivalent). Seen or implied | B1 |
| | $2(10-7) + 2(13-8)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3}{5}$ | Substituting **both** $x = 10$ and $y = 13$ into a valid differentiation to find a value for $\frac{dy}{dx}$ | M1 |
| | $y - 13 = -\frac{3}{5}(x-10)$ | $y - 13 = (\text{their gradient})(x-10)$ | M1 |
| | $3x + 5y - 95 = 0$ | $3x + 5y - 95 = 0$ o.e. | A1 |
| | | | **[4]** |
| (c) **Way 3** | $10x + 13y - 7(x+10) - 8(y+13) + 79 = 0$ | $10x + 13y - 7(x+10) - 8(y+13) + 79 = 0$ | B1 |
| | | $10x + 13y - 7(x+10) - 8(y+13) + c = 0$ where $c$ is any value $< 113$ | M2 |
| | $3x + 5y - 95 = 0$ | $3x + 5y - 95 = 0$ o.e. | A1 |
| | | | **[4]** |
| | | | **8** |

| | | Question 3 Notes |
|---|---|---|
| (a) | **M1** | Allow for $\{PQ =\}\ \sqrt{(7-10)^2+(8-13)^2}$ or for $\{PQ =\}\ \sqrt{3^2+5^2}$ . Can be implied by answer. |
| | **A1** | Need to see $\sqrt{34}$ . You can ignore subsequent work so $\sqrt{34}$ followed by 5.83 earns M1 A1, but $\{PQ =\}\ \sqrt{3^2+5^2}=5.83$, with no exact value for the answer given, earns M1A0. Allow $\pm\sqrt{34}$ this time.<br>NB Some use equation of circle to find this distance Achieving $\sqrt{34}$ gets M1A1<br>Others find half of their $\pm\sqrt{34}$ . Do not isw here as it is an error – confusing *d* with diameter. Give M1A0 |
| **(b)** | **M1** | Either of the correct approaches for equation of circle (as shown on scheme) |
| | **A1** | Correct equation (two are shown and any correct equivalent is acceptable) |
| **(c)** | | A correct start to finding the gradient of the tangent (see each scheme) |
| | **B1** | Complete method for finding the gradient of the tangent (see each scheme) Where implicit differentiation has been used the only slips allowed here should be sign slips. |
| | **1st M1** | Correct attempt at line equation for tangent at correct point (10, 13) with **their tangent** gradient. If the $y = mx + c$ method is used to find the equation, this M1 is earned at the point where the *x*- and *y*-values are substituted to find *c* e.g. $13 = -3/5 \times 10 + c$ |
| | **2nd M1** | |
| | **A1** | Accept any correct answer of the required format; so integer multiple of $3x+5y-95=0$ or $3x-95+5y=0$ or $-3x-5y+95=0$ (must include "=0") e.g. $6x + 10y - 190 = 0$ earns A1<br>Also allow $5y + 3x - 95=0$ etc |
| | **Common error** | $\frac{dy}{dx}=2(x-7)+2(y-8)=6+10=16$ so $(y-13)=16(x-10)$ is marked B0 M0 M1 A0 (Way 2) |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.** | $\text{f}(x) = 6x^3 + 13x^2 - 4$ | | |
| (a) | $\text{f}\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^3 + 13\left(-\frac{3}{2}\right)^2 - 4 = 5$ | Attempting $\text{f}\left(-\frac{3}{2}\right)$ or $\text{f}\left(\frac{3}{2}\right)$ | M1 |
| | | 5 | A1 cao |
| | | | **[2]** |
| (b) | $\text{f}(-2) = 6(-2)^3 + 13(-2)^2 - 4$ | Attempts $\text{f}(-2)$. | M1 |
| | $= 0$, **and** so $(x+2)$ is a factor. | $\text{f}(-2) = 0$ with no sign or substitution errors **and for conclusion.** | A1 |
| | | | **[2]** |
| (c) | $\text{f}(x) = \{(x+2)\}(6x^2 + x - 2)$ | | M1 A1 |
| | $= (x+2)(2x-1)(3x+2)$ | | M1 A1 |
| | | | **[4]** |
| | | | **8** |

**Question 4 Notes**

| | | |
|---|---|---|
| | **Note** | **Long division scores no marks in part (a). The <u>remainder theorem</u> is required.** |
| (a) | **M1** | Attempting $\text{f}\left(-\frac{3}{2}\right)$ or $\text{f}\left(\frac{3}{2}\right)$. $6\left(-\frac{3}{2}\right)^3 + 13\left(-\frac{3}{2}\right)^2 - 4$ or $6\left(\frac{3}{2}\right)^3 + 13\left(\frac{3}{2}\right)^2 - 4$ is sufficient |
| | **A1** | 5 cao |
| (b) | **M1** | Attempting $\text{f}(-2)$. (This is **not** given for f(2)) |
| | **A1** | Must correctly show $\text{f}(-2) = 0$ **and** give a conclusion ***in part (b) only***. No simplification of terms is required here. |
| | **Note** | Stating "hence factor" or "it is a factor" or a "tick" or "QED" are possible conclusions. Also a conclusion can be implied from a <u>preamble</u>, eg: "If $\text{f}(-2) = 0$, $(x+2)$ is a factor…." **Long division scores no marks in part (b). The <u>factor theorem</u> is required.** |
| (c) | **1st M1** | Attempting to divide by $(x+2)$ leading to a quotient which is quadratic with at least two terms beginning with first term of $\pm 6x^2$ + linear or constant term. Or $\text{f}(x) = (x+2)(\pm 6x^2 + \text{linear and/or constant term})$ (This may be seen in part (b) where candidates did not use factor theorem and might be referred to here) |
| | **1st A1** | $(6x^2 + x - 2)$ seen as quotient or as factor. If there is an error in the division resulting in a remainder give A0, but allow recovery to gain next two marks if $(6x^2 + x - 2)$ is used |
| | **2nd M1** | For a ***valid*** attempt to factorise **their** three term quadratic. |
| | **A1** | $(x+2)(2x-1)(3x+2)$ and needs all three factors on the same line. Ignore subsequent work (such as a **solution** to a quadratic equation). |
| | **Special cases** | **Calculator methods:** Award M1A1M1A1 for correct answer $(x+2)(2x-1)(3x+2)$ with no working. Award M1A0M1A0 for either $(x+2)(2x+1)(3x+2)$ or $(x+2)(2x+1)(3x-2)$ or $(x+2)(2x-1)(3x-2)$ with no working. (At least one bracket incorrect) Award M1A1M1A1 for $x = -2, \frac{1}{2}, -\frac{2}{3}$ followed by $(x+2)(2x-1)(3x+2)$. Award M0A0M0A0 for a candidate who writes down $x = -2, \frac{1}{2}, -\frac{2}{3}$ giving no factors. Award M1A1M1A1 for $6(x+2)(x-\frac{1}{2})(x+\frac{2}{3})$ or $2(x+2)(x-\frac{1}{2})(3x+2)$ or equivalent. Award SC: M1A0M1A0 for $x = -2, \frac{1}{2}, -\frac{2}{3}$ followed by $(x+2)(x-\frac{1}{2})(x+\frac{2}{3})$. |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **5.** | (a) $(2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $f(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2$ | | |
| (a) **Way 1** | First term of 16 in their final series | | B1 |
| | At least one of $({}^4C_1 \times ... \times x)$ or $({}^4C_2 \times ... \times x^2)$ | | M1 |
| | $= (16) - 288x + 1944x^2$ | At least one of $-288x$ or $+1944x^2$ | A1 |
| | | Both $-288x$ **and** $+1944x^2$ | A1 |
| | | | **[4]** |
| (a) **Way 2** | $(2-9x)^4 = (4-36x+81x^2)(4-36x+81x^2)$ | | |
| | | First term of 16 in their final series | B1 |
| | $= 16 - 144x + 324x^2 - 144x + 1296x^2 + 324x^2$ | Attempts to multiply a 3 term quadratic by the same 3 term quadratic to achieve either 2 terms in $x$ or at least 2 terms in $x^2$. | M1 |
| | $= (16) - 288x + 1944x^2$ | At least one of $-288x$ or $+1944x^2$ | A1 |
| | | Both $-288x$ **and** $+1944x^2$ | A1 |
| | | | **[4]** |
| (a) **Way 3** | $\left\{(2-9x)^4 =\right\}\ 2^4\left(1-\frac{9}{2}x\right)^4$ | First term of 16 in final series | B1 |
| | $= 2^4\left(1 + \underline{4\left(-\frac{9}{2}x\right) + \frac{4(3)}{2}\left(-\frac{9}{2}x\right)^2} + ...\right)$ | At least one of $\underline{(4\times...\times x) \text{ or } \left(\frac{4(3)}{2}\times...\times x^2\right)}$ | M1 |
| | $= (16) - 288x + 1944x^2$ | At least one of $-288x$ or $+1944x^2$ | A1 |
| | | Both $-288x$ **and** $+1944x^2$ | A1 |
| | | | **[4]** |
| | Parts (b), (c) and (d) may be marked together | | |
| (b) | $A = "16"$ | Follow through their value from (a) | B1ft |
| | | | **[1]** |
| (c) | $\left\{(1+kx)(2-9x)^4\right\} = (1+kx)\left(16 - 288x + \left\{1944x^2 + ...\right\}\right)$ | *May be seen in part (b) or (d) and can be implied by work in parts (c) or (d).* | M1 |
| | $x$ terms: $-288x + 16kx = -232x$ | | |
| | giving, $16k = 56 \Rightarrow \underline{k = \frac{7}{2}}$ | $\underline{k = \frac{7}{2}}$ | A1 |
| | | | **[2]** |
| (d) | $x^2$ terms: $1944x^2 - 288kx^2$ | | |
| | So, $B = 1944 - 288\left(\frac{7}{2}\right)$; $= 1944 - 1008 = 936$ | See notes | M1 |
| | | 936 | A1 |
| | | | **[2]** |
| | | | **9** |

| | Question 5 Notes | |
|---|---|---|
| (a) Ways 1 and 3 | **B1 cao** | 16 |
| | **M1** | Correct binomial coefficient associated with correct power of *x i.e* $\left({}^4C_1 \times ... \times x\right)$ or $\left({}^4C_2 \times ... \times x^2\right)$<br>They may have 4 and 6 or 4 and $\frac{4(3)}{2}$ or even $\binom{4}{1}$ and $\binom{4}{2}$ as their coefficients. Allow missing signs and brackets for the M marks. |
| | **1st A1** | At least one of $-288x$ or $+1944x^2$ (allow +- 288*x*) |
| | **2nd A1** | Both $-288x$ **and** $+1944x^2$ (May list terms separated by commas) Also full marks for correct answer with no working here. Again allow +- 288*x* |
| | **Note** | If the candidate then divides their final correct answer through by 8 or any other common factor then isw and mark correct series when first seen. So (a) B1M1A1A1 .It is likely that this approach will be followed by (b) B0, (c) M1A0, (d) M1A0 if they continue with their new series e.g. $2-36x+283x^2+...$ (Do not ft the value 2 as a mark was awarded for 16) |
| **Way 2b** | **Special Case** | **Slight Variation on the solution given in the scheme**<br>$(2-9x)^4=(2-9x)(2-9x)(4-36x+81x^2)$<br>$=(2-9x)(8-108x+486x^2+...)$<br>$=16-216x+972x^2-72x+972x^2$ — First term of 16: **B1**; Multiplies out to give either 2 terms in *x* or 2 terms in $x^2$.: **M1**<br>$=(16)-288x+1944x^2+...$ — At least one of $-288x$ or $+1944x^2$: **A1**; Both $-288x$ **and** $+1944x^2$: **A1** |
| | | **Parts (b), (c) and (d) may be marked together.** |
| (b) | **B1ft** | Must **identify** $A=16$ or $A=$*their* constant term found in part (a). Or may write just 16 if this is clearly their answer to part (b). If they expand their series and have 16 as first term of a series it is not sufficient for this mark. |
| (c) | **M1** | Candidate shows intention to multiply (1+*kx*) by part of their series from (a)<br>e.g. Just $(1+kx)\left(16-288x+...\right)$ or $(1+kx)\left(16-288x+1944x^2+...\right)$ are fine for M1. |
| | **Note** | This mark can also be implied by candidate multiplying out to find **two terms** (or coefficients) in *x*. i.e. f.t. their $-288x+16kx$ N.B. $-288kx=-232x$ with no evidence of brackets is M0 – allow copying slips, or use of factored series, as this is a method mark |
| | **A1** | $k=\frac{7}{2}$ o.e. so 3.5 is acceptable |
| (d) | **M1** | Multiplies out their $(1+kx)\left(16-288x+1944x^2+...\right)$ to give **exactly** two terms (or coefficients) in $x^2$ and attempts to find *B* using **these two** terms and a numerical value of *k*. |
| | **A1** | 936 |
| | **Note** | Award A0 for $B=936x^2$<br>But allow A1 for $B=936x^2$ **followed by** $B=936$ and treat this as a correction<br>Correct answers in parts (c) and (d) with no method shown may be awarded full credit. |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **6.** | $1-2\cos\left(\theta-\frac{\pi}{5}\right)=0\,;\ -\pi<\theta\leqslant\pi$ | | |
| (i) | $\cos\left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$ | Rearranges to give $\cos\left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$ or $-\frac{1}{2}$ | M1 |
| | $\theta=\left\{-\frac{2\pi}{15},\ \frac{8\pi}{15}\right\}$ | At least one of $-\frac{2\pi}{15}$ or $\frac{8\pi}{15}$ or $-24^\circ$ or $96^\circ$ or awrt 1.68 or awrt -0.419 | A1 |
| | | Both $-\frac{2\pi}{15}$ and $\frac{8\pi}{15}$ | A1 |
| | | | **[3]** |
| **NB Misread** | **Misreading** $\frac{\pi}{5}$ as $\frac{\pi}{6}$ or $\frac{\pi}{3}$ (or anything else)– treat as misread so M1 A0 A0 is maximum mark | | |
| | $4\cos^2 x+7\sin x-2=0\,,\ 0\leqslant x<360^\circ$ | | |
| (ii) | $4(1-\sin^2 x)+7\sin x-2=0$ | Applies $\cos^2 x=1-\sin^2 x$ | M1 |
| | $4-4\sin^2 x+7\sin x-2=0$ | | |
| | $4\sin^2 x-7\sin x-2=0$ | Correct 3 term, $4\sin^2 x-7\sin x-2\ \{=0\}$ | A1 oe |
| | $(4\sin x+1)(\sin x-2)\ \{=0\}\ ,\ \sin x=...$ | Valid attempt at solving and $\sin x=...$ | M1 |
| | $\sin x=-\frac{1}{4}\,,\quad\{\sin x=2\}$ | $\sin x=-\frac{1}{4}$ (See notes.) | A1 cso |
| | $x=\text{awrt}\{194.5,\ 345.5\}$ | At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0 | A1ft |
| | | awrt 194.5 **and** awrt 345.5 | A1 |
| | | | **[6]** |
| | | | **9** |
| **NB Misread** | **Writing equation as** $4\cos^2 x-7\sin x-2=0$ with a sign error should be marked by applying the scheme as **it simplifies** the solution (do not treat as misread) Max mark is 3/6 | | |
| | $4(1-\sin^2 x)-7\sin x-2=0$ | | **M1** |
| | $4\sin^2 x+7\sin x-2=0$ | | **A0** |
| | $(4\sin x-1)(\sin x+2)\ \{=0\}\ ,\ \sin x=...$ | Valid attempt at solving and $\sin x=...$ | **M1** |
| | $\sin x=+\frac{1}{4}\,,\quad\{\sin x=-2\}$ | $\sin x=\frac{1}{4}$ (See notes.) | **A0** |
| | $x=\text{awrt}165.5$ | | **A1ft** |
| | Incorrect answers | | **A0** |

| Question 6 Notes | | |
|---|---|---|
| (i) | **M1** | Rearranges to give $\cos\left(\theta - \frac{\pi}{5}\right) = \pm\frac{1}{2}$ |
| | **Note** | M1 can be implied by seeing either $\frac{\pi}{3}$ or $60^\circ$ as a result of taking $\cos^{-1}(...)$. |
| | **A1** | Answers **may be in degrees or radians** for this mark and may have just one correct answer<br>Ignore mixed units in working if correct answers follow (recovery) |
| | **A1** | Both answers correct and in radians as multiples of π $-\frac{2\pi}{15}$ and $\frac{8\pi}{15}$<br>Ignore EXTRA solutions outside the range $-\pi < \theta \le \pi$ but lose this mark for extra solutions in this range. |
| (ii) | **1st M1** | Using $\cos^2 x = 1 - \sin^2 x$ on the given equation. [Applying $\cos^2 x = \sin^2 x - 1$, scores M0.] |
| | **1st A1** | Obtaining a correct three term equation eg. either $4\sin^2 x - 7\sin x - 2\ \{= 0\}$<br>or $-4\sin^2 x + 7\sin x + 2\ \{= 0\}$ or $4\sin^2 x - 7\sin x = 2$ or $4\sin^2 x = 7\sin x + 2$, etc. |
| | **2nd M1** | For a valid attempt at solving a 3TQ quadratic in sine. Methods include factorization, quadratic formula, completion of the square (unlikely here) and calculator. (See notes on page 6 for general principles on awarding this mark) Can use any variable here, *s*, *y*, *x* or $\sin x$, and an attempt to find at least one of the solutions for sin*x*. This solution may be outside the range for sin*x* |
| | **2nd A1** | $\sin x = -\frac{1}{4}$ BY A CORRECT SOLUTION ONLY UP TO THIS POINT**.** Ignore extra answer of $\sin x = 2$, but penalise if candidate states an incorrect result. e.g. $\sin x = -2$. |
| | **Note** | $\sin x = -\frac{1}{4}$ can be implied by later correct working if no errors are seen. |
| | **3rd A1ft** | At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0. This is a limited follow through.<br>Only follow through on the error $\sin x = \frac{1}{4}$ and allow for 165.5 special case (as this is equivalent work) This error is likely to earn M1A1M1A0A1A0 so 4/6 or M1A0M1A0A1A0 if the quadratic had a sign slip. |
| | **4thA1**<br>**Note** | awrt 194.5 **and** awrt 345.5<br>If there are any EXTRA solutions inside the range $0 \leqslant x < 360^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final A1 mark.<br>Ignore EXTRA solutions outside the range $0 \leqslant x < 360^\circ$. |
| | Special Cases | Rounding error Allow M1A1M1A1A1A0 for those who give two correct answers but wrong accuracy e.g. awrt 194, 346 (Remove final A1 for this error)<br>Answers in radians:– **lose final** mark so either or both of 3.4, 6.0 gets A1ftA0<br>It is possible to earn M1A0A1A1 on the final 4 marks if an error results fortuitously in sin*x* = -1/4 then correct work follows. |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7.** (a) | $\left\{\int\left(3x - x^{\frac{3}{2}}\right)dx\right\} = \frac{3x^2}{2} - \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}\ \{+c\}$ | Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$ | M1 |
| | | At least one term correctly integrated | A1 |
| | | Both terms correctly integrated | A1 |
| | | | **[3]** |
| (b) | $0 = 3x - x^{\frac{3}{2}} \Rightarrow 0 = 3 - x^{\frac{1}{2}}$ or $0 = x\left(3 - x^{\frac{1}{2}}\right) \Rightarrow x = ...$ | Sets $y = 0$, in order to find the correct $x^{\frac{1}{2}} = 3$ or $x = 9$ | M1 |
| | $\left\{\text{Area}(S) = \left[\frac{3x^2}{2} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^9\right\}$ | | |
| | $= \left(\frac{3(9)^2}{2} - \left(\frac{2}{5}\right)(9)^{\frac{5}{2}}\right) - \{0\}$ | Applies the limit 9 on an integrated function with **no wrong lower limit**. | ddM1 |
| | $\left\{= \left(\frac{243}{2} - \frac{486}{5}\right) - \{0\}\right\} = \frac{243}{10}$ or 24.3 | $\frac{243}{10}$ or 24.3 | A1 oe |
| | | | **[3]** |
| | | | **6** |

**Question 7 Notes**

| | | |
|---|---|---|
| (a) | **M1** | Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$ |
| | **1st A1** | At least one term correctly integrated. Can be simplified or un-simplified but power must be simplified. Then isw. |
| | **2nd A1** | Both terms correctly integrated. Can be un-simplified (as in the scheme) but the $n$+1 in each denominator and power should be a single number. (e.g. 2 – not 1+1) Ignore subsequent work if there are errors simplifying. Ignore the omission of "$+c$". Ignore integral signs in their answer. |
| (b) | **1st M1** | Sets $y = 0$, and reaches the **correct** $x^{\frac{1}{2}} = 3$ or $x = 9$ (isw if $x^{\frac{1}{2}} = 3$ is followed by $x = \sqrt{3}$)<br>Just seeing $x = \sqrt{3}$ without the correct $x^{\frac{1}{2}} = 3$ gains M0. May just see $x = 9$.<br>Use of trapezium rule to find area is M0A0 as hence implies integration needed. |
| | **ddM1** | This mark is dependent on the two previous method marks and needs both to have been awarded. Sees the limit **9** substituted in an integrated function. (Do not follow through their value of $x$) Do not need to see MINUS 0 but if another value is used as lower limit – this is M0.<br>This mark may be implied by 9 in the limit and a correct answer. |
| | **A1** | $\frac{243}{10}$ or 24.3 |
| | **Common Error** | **Common Error** $0 = 3x - x^{\frac{3}{2}} \Rightarrow x^{\frac{1}{2}} = 3$ *so* $x = \sqrt{3}$<br>**Then** uses limit $\sqrt{3}$ etc gains M1 M0 A0 so 1/3 |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 8(i) | Two Ways of answering the question are given in part (i) | | |
| **Way 1** | $\log_3\left(\dfrac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\dfrac{a-2}{3b+1}\right) = 1$ | Applying the subtraction law of logarithms | M1 |
| | $\dfrac{3b+1}{a-2} = 3^{-1}\left\{=\dfrac{1}{3}\right\}$ or $\left(\dfrac{a-2}{3b+1}\right) = 3$ | Making a correct connection between log base 3 and 3 to a power. | M1 |
| | $\{9b+3 = a-2 \Rightarrow\}\ b = \dfrac{1}{9}a - \dfrac{5}{9}$ | $b = \dfrac{1}{9}a - \dfrac{5}{9}$ or $b = \dfrac{a-5}{9}$ | A1 oe |
| | | | **[3]** |
| | In **Way 2** a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs | | |
| (i) **Way 2** | Either $\log_3(3b+1) - \log_3(a-2) = -\log_3 3$ or $\log_3(3b+1) + \log_3 3 = \log_3(a-2)$ | | 2nd M1 |
| | $\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\dfrac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$ | | 1st M1 |
| | $\{3b+1 = \dfrac{a-2}{3}\}\ b = \dfrac{1}{9}a - \dfrac{5}{9}$ | | A1 |
| | | | **[3]** |
| | Five Ways of answering the question are given in part (ii) | | |
| (ii) | $32(2^{2x}) - 7(2^x) = 0$ | Deals with power 5 correctly giving ×32 | M1 |
| **Way 1** **See also common approach below in notes** | So, $2^x = \dfrac{7}{32}$ | $2^x = \dfrac{7}{32}$ or $y = \dfrac{7}{32}$ or awrt 0.219 | A1 oe |
| | $x\log 2 = \log\left(\dfrac{7}{32}\right)$ or $x = \dfrac{\log\left(\frac{7}{32}\right)}{\log 2}$ or $x = \log_2\left(\dfrac{7}{32}\right)$ | A valid method for solving $2^x = \frac{7}{32}$ Or $2^x = k$ to achieve $x = ...$ | dM1 |
| | $x = -2.192645...$ | awrt $-2.19$ | A1 |
| | | | **[4]** |
| | | | |
| | Begins with $2^{2x+5} = 7(2^x)$ (for Way 2 and Way 3) (see notes below) | | |
| (ii) **Way 2** | $(2x+5)\log 2 = \log 7 + x\log 2$ | Correct application of **either** the power law **or** addition law of logarithms | M1 |
| | | **Correct result** after applying the power **and** addition laws of logarithms. | A1 |
| | $2x\log 2 + 5\log 2 = \log 7 + x\log 2$ <br> $\Rightarrow\ x = \dfrac{\log 7 - 5\log 2}{\log 2}$ | Multiplies out, collects $x$ terms to achieve $x = ...$ | dM1 |
| | $x = -2.192645...$ | awrt $-2.19$ | A1 |
| | | | **[4]** |
| (ii) **Way 3** | $2x + 5 = \log_2 7 + x$ | Evidence of $\log_2$ and either $2^{2x+5} \rightarrow 2x+5$ or $7(2^x) \rightarrow \log_2 7 + \log_2(2^x)$ | M1 |
| | | $2x + 5 = \log_2 7 + x$ oe. | A1 |
| | $2x - x = \log_2 7 - 5$ <br> $\Rightarrow\ x = \log_2 7 - 5$ | Collects $x$ terms to achieve $x = ...$ | dM1 |
| | $x = -2.192645...$ | awrt $-2.19$ | A1 |
| | | | **[4]** |

| | | |
|---|---|---|
| (ii) **Way 4** | $2^{2x+5} = 7(2^x) \Rightarrow 2^{x+5} = 7$ | |
| | $x + 5 = \log_2 7$ or $\dfrac{\log 7}{\log 2}$ — Evidence of $\log_2$ and either $2^{x+5} \to x+5$ or $7 \to \log_2 7$ | M1 |
| | $x + 5 = \log_2 7$ oe. | A1 |
| | $x = \log_2 7 - 5$ — Rearranges to achieve $x = ...$ | dM1 |
| | $x = -2.192645...$ — awrt $-2.19$ | A1 |
| | | **[4]** |
| **Way 5 (similar to Way 3)** | $2^{2x+5} = 2^{\log_2 7}(2^x)$ — 7 is replaced by $2^{\log_2 7}$ | M1 |
| | $2x + 5 = \log_2 7 + x$ — $2x + 5 = \log_2 7 + x$ oe. | A1 |
| | $2x - x = \log_2 7 - 5$ $\Rightarrow x = \log_2 7 - 5$ — Collects $x$ terms to achieve $x = ...$ | dM1 |
| | $x = -2.192645...$ — awrt $-2.19$ | A1 |
| | | **[4]** |
| | | **7** |

| | | **Question 8 Notes** |
|---|---|---|
| (i) | **1st M1** | Applying either the addition or subtraction law of logarithms correctly to combine any **two** log terms into **one** log term. |
| | **2nd M1** | For making a correct connection between log base 3 and 3 to a power. |
| | **A1** | $b = \frac{1}{9}a - \frac{5}{9}$ or $b = \frac{a-5}{9}$ o.e. e.g. Accept $b = \frac{1}{3}\left(\frac{a}{3} - \frac{5}{3}\right)$ but not $b = \frac{a-2}{9} - \frac{3}{9}$ nor $b = \frac{\left(\frac{a}{3} - \frac{5}{3}\right)}{3}$ |
| (ii) | **1st M1** | First step towards solution – an equation with one side or other correct or one term dealt with correctly (see five* possible methods above) |
| | **1st A1** | Completely correct first step – giving a correct equation as shown above |
| | **dM1** | Correct complete method (all log work correct) and working to reach $x$ = in terms of logs reaching a correct expression or one where the only errors are slips solving linear equations |
| | **2nd A1** | Accept answers which round to -2.19 If a second answer is also given this becomes A0 |
| | **Special Case in (i)** | Writes $\frac{\log_3(3b+1)}{\log_3(a-2)} = -1$ and proceeds to $\frac{3b+1}{a-2} = 3^{-1}\left\{=\frac{1}{3}\right\}$ and to correct answer- Give M0M1A1 (special case) |
| | **Common approach to part (ii)** | Let $2^x = y$ Treat this as **Way 1** They get $32y^2 - 7y = 0$ for M1 and need to reach $y = \frac{7}{32}$ for A1. Then back to **Way 1** as before. Any letter may be used for the new variable which I have called $y$. If they use $x$ and obtain $x = \frac{7}{32}$, this may be awarded M1A0M0A0. Those who get $y^2 - 7y + 32 = 0$ or $y^7 - 7y = 0$ will be awarded M0,A0,M0,A0 |
| | **Common Present-ation of Work in ii** | **Many begin with** $\log\left(2^{2x+5}\right) - \log\left(7(2^x)\right) = 0$. It is possible to reach this in two stages correctly so do not penalise this and award the full marks if they continue correctly as in **Way 2**. If however the solution continues with $(2x+5)\log 2 - x\log 14 = 0$ or with $(2x+5)\log 2 - 7x\log 2 = 0$ (both incorrect) then they are awarded M1A0M0A0 just getting credit for the $(2x + 5)$ log2 term. |
| | **Note** | N.B. The answer (+)2.19 results from "algebraic errors solving linear equations" leading to $2^x = \frac{32}{7}$ and gets M1A0M1A0 |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **9.** (a) | $\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right)$; $= \frac{\pi x^2}{3}$ | $\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360}\times \pi x^2$ simplified or un-simplified | M1 |
| | | $\frac{\pi x^2}{3}$ | A1 |
| | | | **[2]** |
| | Parts (b) and (c) may be marked together | | |
| (b) | $\{A =\}\ \frac{1}{2}x^2\sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$ | Attempt to sum 3 areas (at least one correct) | M1 |
| | | Correct expression for at least two terms of $A$ | A1 |
| | $1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$<br>$\Rightarrow y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$ * | Correct proof. | A1 * |
| | | | **[3]** |
| (c) | $\{P =\}\ x + x\theta + y + 2x + y\ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$ | Correct expression in $x$ and $y$ for their $\theta$ measured in rads | B1ft |
| | $...2y = \ + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$ | Substitutes expression from (b) into $y$ term. | M1 |
| | $P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$ | | |
| | $\Rightarrow P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$ * | Correct proof. | A1 * |
| | | | **[3]** |
| | Parts (d) and (e) should be marked together | | |
| (d) | $\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}$; $= 0$ | $\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$ | M1 |
| | | Correct differentiation (need not be simplified). | A1; |
| | | Their $P' = 0$ | M1 |
| | $\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ $(= 16.63392808...)$ | $\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied) | A1 |
| | $\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..$ (m) | awrt 120 | A1 |
| | | | **[5]** |
| (e) | $\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow$ Minimum | Finds $P''$ **and** considers sign. | M1 |
| | | $\frac{2000}{x^3}$ (need not be simplified) and $> 0$ and conclusion.<br>Only follow through on a correct $P''$ and $x$ in range $10 < x < 25$. | A1ft |
| | | | **[2]** |
| | | | **15** |

| | | Question 9 Notes |
|---|---|---|
| (a) | M1 | Attempts to use Area$(FEA) = \frac{1}{2}x^2 \times \frac{2\pi}{3}$ (using radian angle) or $\frac{120}{360}\times \pi x^2$ (using angle in degrees) |
| | A1 | $\frac{\pi x^2}{3}$ cao (Must be simplified and be their answer in part (a)) Answer only implies M1A1. |
| | | N.B. Area$(FEA) = \frac{1}{2}x^2 \times 120$ is awarded M0A0 |
| (b) | M1 | An attempt to sum 3 " areas" consisting of rectangle, triangle and sector (allow slips even in dimensions) but **one area** should be correct |
| | 1st A1 | Correct expression for **two** of the **three** areas listed above. Accept any correct equivalents e.g. two correct from $\frac{1}{2}x^2\sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{4}x^2\sqrt{3}$, $\frac{1}{2}\times\frac{2}{3}\pi x^2$, $2xy$ |
| | 2nd A1* | This is a given answer which should be stated and should be achieved without error so all three areas must have been correct and their sum put equal to 1000 and an intermediate step of rearrangement should be present. |
| (c) | B1ft | Correct expression for $P$ from arc length, length $AB$ and three sides of rectangle in terms of both $x$ and $y$ with $2y$ (or $y + y$), $3x$ (or $x + 2x$) (or $x + x + x$), and $x\theta$ clearly listed . Allow addition after substitution of $y$. |
| | | NB $\theta = \frac{2\pi}{3}$ but allow use of their consistent $\theta$ in radians (usually $\theta = \frac{\pi}{3}$ ) from parts (a) and (b) for this mark. $120x$ or $60x$ do not get this mark. |
| | M1 | Substitutes $y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$ or their unsimplified attempt at $y$ from earlier (allow slips e.g. sign slips) into $2y$ term. |
| | A1* | This is a given answer which should be stated and should be achieved without error |
| (d) | 1st M1 | Need to see at least $\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$ |
| | 1st A1 | Correct differentiation of both terms (need not be simplified) Not follow through. Allow any correct equivalent. |
| | | e.g. $\frac{dP}{dx} = -1000x^{-2} + \frac{\pi}{3} + 3 - \frac{\sqrt{3}}{4}$ Also allow $\frac{dP}{dx} = -1000x^{-2}$ + *awrt* 3.61 |
| | | Check carefully as there are many correct equivalents and some have two terms in $x\pi$ to differentiate obtaining for example $\frac{2\pi}{3} - \frac{8\pi}{24}$ instead of $\frac{\pi}{3}$ |
| | 2nd M1 | Setting their $\frac{dP}{dx} = 0$. Do not need to find $x$, but if inequalities are used this mark cannot be gained until candidate states or uses a value of $x$ without inequalities. May not be explicit but may be implied by correct working and value or expression for $x$ . May result in $x^2 < 0$ so M1A0 |
| | 2nd A1 | There is no requirement to write down a value for $x$, so this mark may be implied by a correct value for $P$. It may be given for a correct expression or value for $x$ of 16.6, 16.7 or 17 |
| | 3rd A1 | Allow answers wrt 120 but not 121 |
| (e) | M1 | Finds $P''$ **and** considers sign. Follow through **correct** differentiation of their $P'$ (not just reduction of power) |
| | A1ft | Need $\frac{2000}{x^3}$ and $> 0$ (or positive value) and conclusion. Only follow through on a correct $P''$ and a value for $x$ in the range $10 < x < 25$ (need not see $x$ substituted but an $x$ should have been found) |
| | | If $P$ is substituted then this is awarded M1 A0 |

| | | |
|---|---|---|
| | **Special case** | (d) Some candidates multiply $P$ by 12 to "simplify" If they write $\frac{\mathrm{d}P}{\mathrm{d}x} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}$ ; $= 0$ then solve they will get the correct $x$ and $P$ They should be awarded M1A0M1A1A1 in part (d). If they then do part (e) writing $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{24000}{x^3} > 0 \Rightarrow$ Minimum They should be awarded M1A0 (so lose 2 marks in all) If they wrote $\frac{\mathrm{d}(12P)}{\mathrm{d}x} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}$ ; $= 0$ etc they could get full marks. |